Election Forensics

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Election Integrity: Competitive and Sound Elections?

- free and fair elections (Norris, 2014) that have correct outcomes
- procedural frauds: things are not done right
- realized frauds: the outcome is adversely affected by malevolent distortions of electors' intentions
- also there can be accidents

Assessment Methods: What is the Evidence?

- observers
- public opinion surveys
- challenges
- recounts
- other statistical approaches with aggregate vote data, e.g., precinct-level eligible voter, registered voter and vote counts

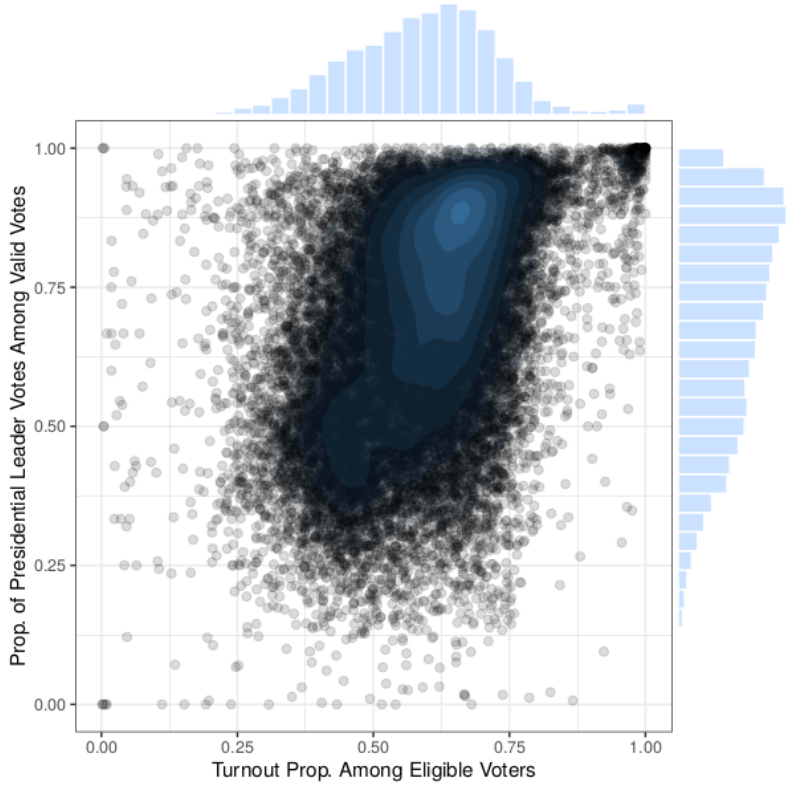
Election Forensics

- election forensics: using statistical methods to determine whether the results of an election accurately reflect the intentions of the electors
- three statistical approaches
  1. identify patterns that look weird: anomalies
  2. test for significant discrepancies between hand counting and other tabulation methods (e.g., parallel vote tabulation, risk-limiting audits)
  3. estimate the prevalence and magnitude of "frauds": how many precincts have frauds and how many votes are fraudulent?
- the `eforensics` model attempts the estimation approach

Anomalies

- various kinds of patterns have been said to be anomalous in aggregated vote data and to suggest (or indicate) that there are election frauds
  1. relationships between precinct turnout and vote shares, and multimodalities in turnout and vote shares scatterplots ("fingerprints of frauds") (Myagkov, Ordeshook and Shaikin, 2009)
  2. digit tests (Hicken and Mebane, 2015)
  3. vote share tests (Rozenas, 2017)
  4. nonsense (Eggers, Garro and Grimmer, 2021)

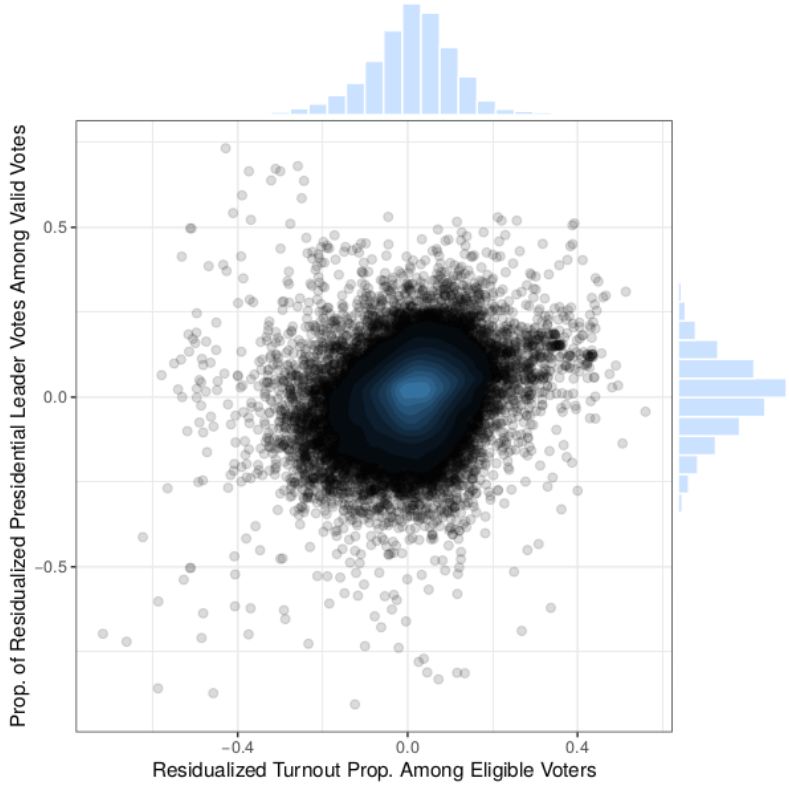
Anomaly Example: Uganda 2011 President



(a) original data

Note: `eforensics`-plot: scatterplot, 2D empirical density and marginal histograms for turnout and leader vote proportions. $n = 23827$ polling stations.

Brittle Anomaly?: Uganda 2011 President



(b) province-residualized data

Note: `eforensics`-plot: scatterplot, 2D empirical density and marginal histograms for turnout and leader vote proportions. $n = 23827$ polling stations.

Estimating "Frauds"

- finite-mixture simulations used to estimate frauds with aggregated turnout and vote data (Klimek, Yegorov, Hanel and Thurner, 2012; Klimek, Jiménez, Hidalgo, Hinteregger and Thurner, 2018)
- a classification approach to estimating frauds using random forests (Zhang, Alvarez and Levin, 2019)
- `eforensics`: a finite-mixture model with Bayesian estimation (Ferrari, Mebane, McAlister and Wu, 2019; Mebane, 2022, 2023)

What eforensics Estimates

- eforensics estimates whether each precinct has "frauds," and for each precinct that has "frauds" it estimates how many votes for that precinct are "fraudulent"
  - I use the expression eforensics-frauds to refer to the kinds of "frauds" produced by the mathematical and statistical formulation of the eforensics model
  - for each precinct (indexed by i), F_{wi} denotes the number of eforensics-fraudulent votes for precinct i; these are votes that were counted for the election leader but, according to the model, should not have been
  - F_{ti} denotes the number of manufactured votes: these are votes counted for the leader but, according to the model, should have been abstentions
  - $F_{wi} - F_{ti}$ is the number of stolen votes: these are votes counted for the leader but, according to the model, should have been counted for opposition candidates, parties or other valid ballot alternatives

Validity of Estimated eforensics-frauds?

- the most important question is whether eforensics-frauds and eforensics-fraudulent votes measure malevolent distortions of electors' intentions
- check how eforensics estimates relate to decisions by election courts
  1. France 2017 National Assembly: 8 of 577 districts annulled ($n = 68760$ *bureaux*)
  2. Mexico 2006 president: 716 of 130,788 *casillas* annulled
  3. Uganda 2006 president: 355 "malpractices" confirmed among 19,786 polling stations
- while the courts' decisions are heavily procedural—they are legal decisions—they often explicitly include quantitative assessments hence refer to alleged realized frauds, and the decisions were widely accepted by the public and by contesting candidates and parties

Table: 2017 *Conseil Constitutionnel* Annulments Regressed on *bureaux* eforensics-fraudulent Votes Aggregated by District

	(B) annulments		
regressor	(e)[b]	(f)[b]	(g)[b]
Intercept	−5.76	−5.77	−5.79
	[−6.55, −4.98]	[−6.59, −5.00]	[−6.52, −4.96]
$F_t/(M+1)$	.795	—	—
	[.640, .974]	—	—
$(F_w - F_t)/(M+1)$	—	3.73	—
	—	[2.97, 4.55]	—
$F_w/(M+1)$	—	—	3.73
	—	—	[3.03, 4.53]
n of districts	302	302	302
AIC[c]	61.0	60.9	61.0

Note: binomial logistic regressions (adjusted for selectivity) of French 2017 National Assembly election *Conseil Constitutionnel* annulments (by district) on *bureaux* eforensics-frauds estimates using second round votes. (e–h) are from binomial logistic regressions for annulment decisions. Upper bounds of $F_t/(M+1)$, $(F_w - F_t)/(M+1)$ and $F_w/(M+1)$ (lower bounds are always zero): 6.12, 1.30 and 7.775 for 2017.

Table: Mexico 2006 President *Casilla* Nullifications Regressed on eforensics-frauds Estimated with *Municipio* Party and *Estado* Fixed Effects

regressor	(a)	(b)	(c)
Intercept	-6.33	-6.32	-6.33
	$[-6.34, -6.31]$	$[-6.34, -6.31]$	$[-6.34, -6.31]$
$F_{ti}/(M_i+1)$	.152	—	—
	[.127, .178]	—	—
$(F_{wi}-F_{ti})/(M_i+1)$	—	.198	—
	—	[.183, .257]	—
$F_{wi}/(M_i+1)$	—	—	.0820
	—	—	[.0955, .122]
AIC	5390.5	5392.0	5389.9

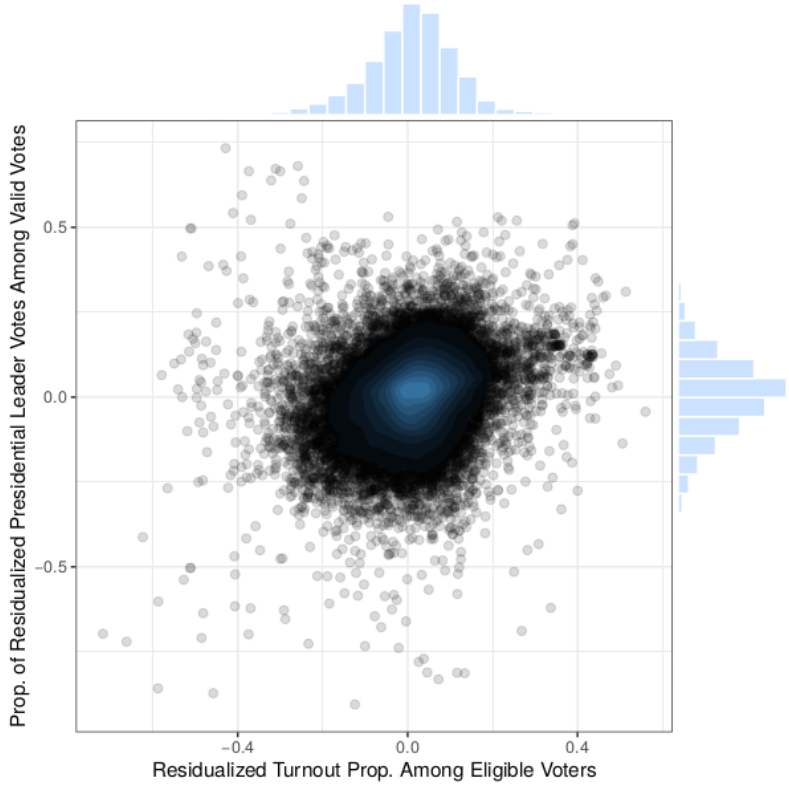
Note: binomial logistic regression (adjusted for selectivity) of TEPJF *casilla* nullifications by *casilla* (coefficient Normal approximation mean and 95% confidence interval based on robust covariance matrices are shown). $n = 26168$. M_i is the margin (vote count difference) between the largest and second-largest vote counts at each *casilla*. Upper bounds of $F_{ti}/(M_i+1)$, $(F_{wi}-F_{ti})/(M_i+1)$ and $F_{wi}/(M_i+1)$ (lower bounds are always zero): 147., 54.3., 201. Product-moment correlation: $\mathrm{cor}(F_{ti}/M_i, (F_{wi}-F_{ti})/M_i) = .905$.

Table: Uganda 2006 President Allegations Regressed on eforensics-fraudulent Votes: Affirmed by Court, Any Allegation

regressor	(a)	(b)	(c)
Intercept	−4.08	−4.19	−4.14
	[−4.19, −3.96]	[−4.33, −4.06]	[−4.26, −4.01]
F_{ti}	.00521	—	—
	[.00261, .00794]	—	—
$F_{wi} - F_{ti}$	—	.0107	—
	—	[.00746, .0139]	—
F_{wi}	—	—	.00418
	—	—	[.00278, .00555]
AIC[a]	3551.8	3517.3	3536.4

Note: "any allegation" is true if any allegation has been affirmed about the polling station. Binomial logistic regression of allegations by polling station (coefficient Normal approximation mean and 95% confidence interval based on robust covariance matrices are shown). [a] AIC is from the regression using the posterior mean of the regressor.
$n = 19750$ polling station units. Upper bounds of F_{ti}, $F_{wi} - F_{ti}$ and F_{wi} (lower bounds are always zero): 319.7, 246.1, 523.4. Product-moment correlation: $\mathrm{cor}(F_{ti}, (F_{wi} - F_{ti})) = .93$.

Brittle Anomaly?: Uganda 2011 President



(b) province-residualized data

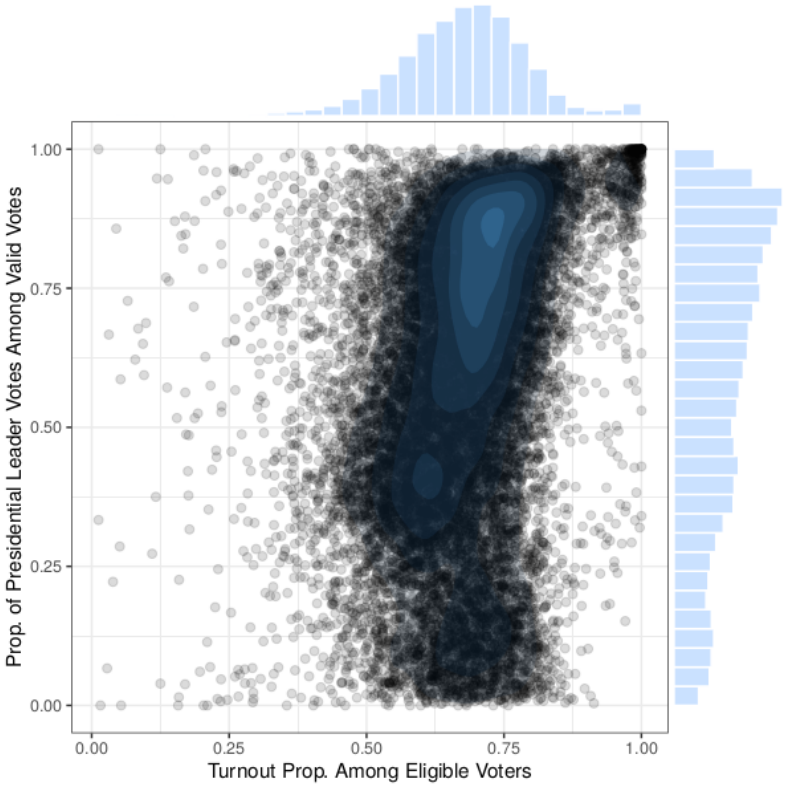
Note: eforensics-plot: scatterplot, 2D empirical density and marginal histograms for turnout and leader vote proportions. $n = 23827$ polling stations.

Table: Uganda 2011 President Election eforensics Estimates, Province Fixed Effects

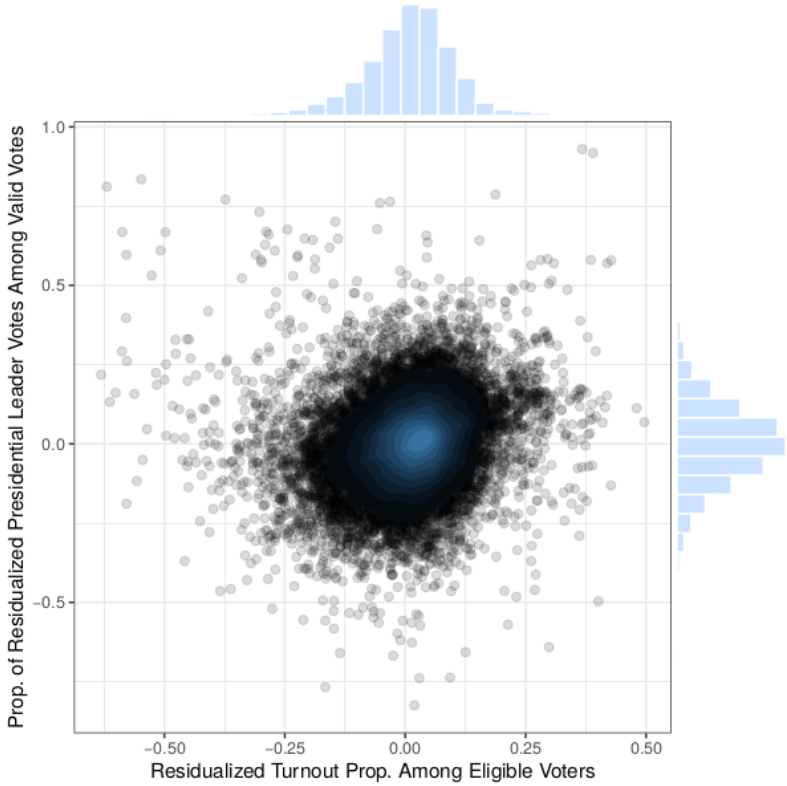
Type	Parameter	Covariate	Mean	lo[a]	up[b]
mixture probabilities	π_1	No Fraud	.786	.771	.802
	π_2	Incremental Fraud	.196	.180	.211
	π_3	Extreme Fraud	.0182	.0165	.0200
units eforensics-fraudulent: (3921 incremental, 447 extreme, 19459 not fraudulent)					
manufactured votes		$F_t = 225494.7$ [202820.2, 247391.9][e]			
incremental manufactured		$F_t = 158839.4$ [136558.9, 178784.1][e]			
extreme manufactured		$F_t = 66655.3$ [64567.6, 69155.7][e]			
total eforensics-fraudulent votes		$F_w = 312556.3$ [280646.5, 342569.0][e]			
incremental total		$F_w = 226407.3$ [195922.6, 253788.1][e]			
extreme total		$F_w = 86149.0$ [83863.6, 89690.5][e]			

Note: selected eforensics model parameter estimates (posterior means and credible intervals). Province fixed effects for turnout, vote choice and eforensics-frauds magnitudes are not shown $n = 23827$ polling station units. Electors, valid votes and votes for the leader: $\sum_{i=1}^{n} N_i = 13875338$; $\sum_{i=1}^{n} V_i = 7928276$; $\sum_{i=1}^{n} W_i = 5436639$. [a] 95% HPD lower bound. [b] 95% HPD upper bound. [e] posterior mean [99.5% credible interval].

eforensics-plots: Uganda 2006 President



(a) 2006 original data



(b) 2006 district-residualized data

Note: eforensics-plots: scatterplots, 2D empirical densities and marginal histograms for turnout and leader vote proportions.
$n = 19750$ polling stations

Table: Uganda 2006 President Election eforensics Estimates, District Fixed Effects

Parameter Estimates

Type	Parameter	Covariate	Mean	lo[a]	up[b]
mixture probabilities	π_1	No Fraud	.746	.735	.759
	π_2	Incremental Fraud	.228	.216	.239
	π_3	Extreme Fraud	.0263	.0239	.0290

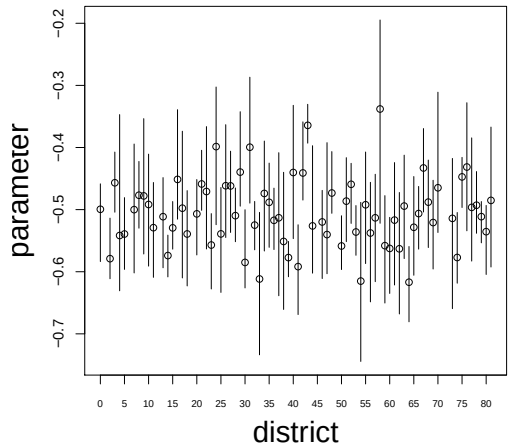
units eforensics-fraudulent: (4092 incremental, 533 extreme, 15125 not fraudulent)

manufactured votes	$F_t = 174188.2\ [159129.1, 183211.2]^e$
incremental manufactured	$F_t = 117059.2\ [104602.2, 124326.0]^e$
extreme manufactured	$F_t = 57129.0\ [54376.8, 59133.0]^e$
total eforensics-fraudulent votes	$F_w = 362062.7\ [333893.5, 378865.2]^e$
incremental total	$F_w = 263620.3\ [239357.9, 278052.2]^e$
extreme total	$F_w = 98442.3\ [94367.4, 102026.5]^e$

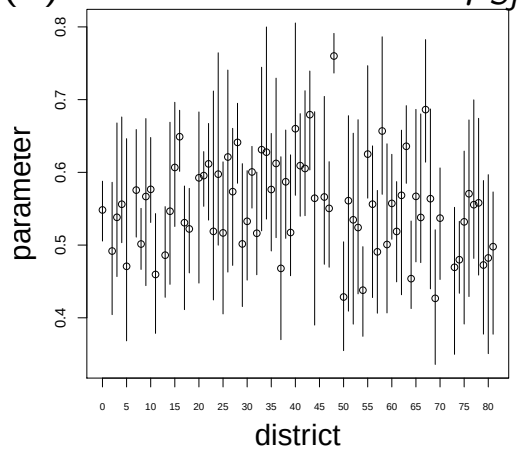
Note: selected eforensics model parameter estimates (posterior means and credible intervals). District fixed effects for turnout, vote choice and eforensics-frauds magnitudes are not shown. $n = 19750$ polling station units. Electors, valid votes and votes for the leader: $\sum_{i=1}^{n} N_i = 10437129$; $\sum_{i=1}^{n} V_i = 6941154$; $\sum_{i=1}^{n} W_i = 4111955$. [a] 95% HPD lower bound. [b] 95% HPD upper bound. [e] posterior mean [99.5% credible interval].

Figure: Uganda 2006 President: eforensics-frauds Magnitude Fixed Effect Parameters

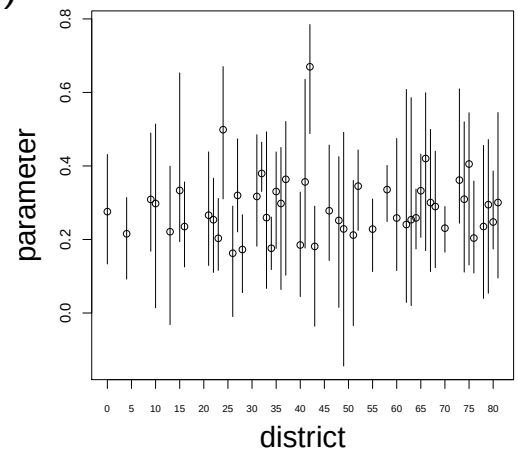
(a) incremental manufactured: ρ_{Mj}



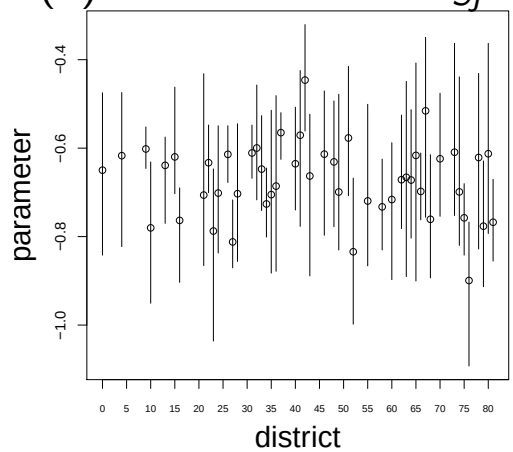
(b) incremental stolen: ρ_{Sj}



(c) extreme manufactured: δ_{Mj}

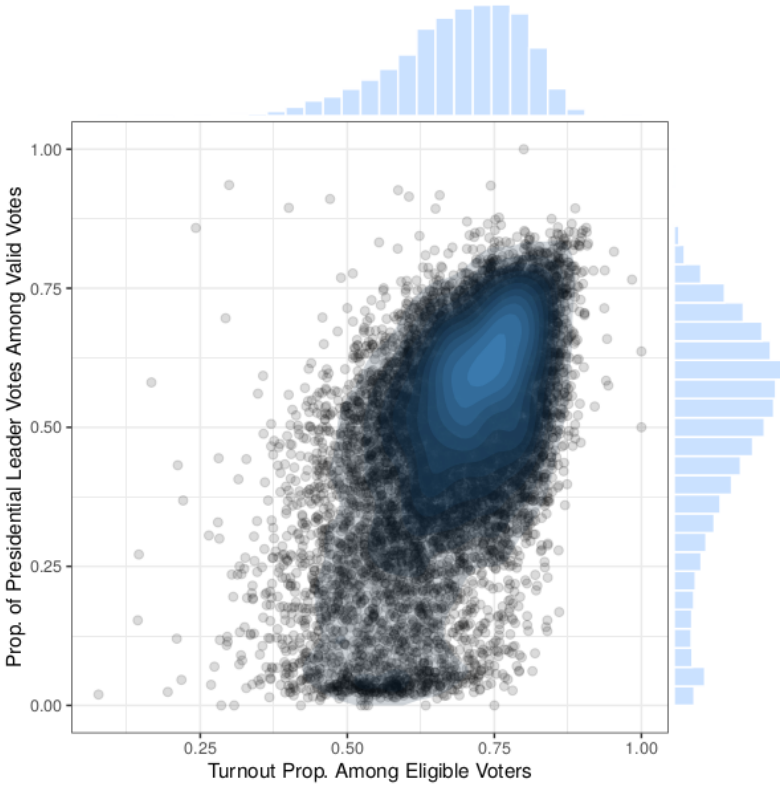


(d) extreme stolen: δ_{Sj}

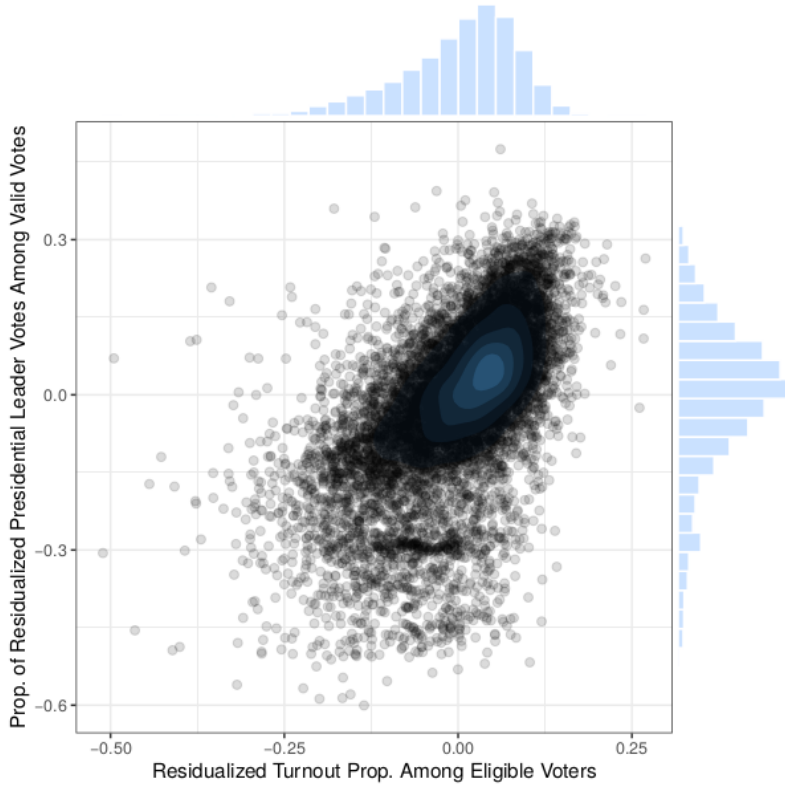


Note: active fixed effects parameters (posterior means and 95% HPD intervals) for frauds magnitude (ρ_{Mj}, ρ_{Sj}, δ_{Mj}, δ_{Sj}) parameters in the eforensics model reported in

eforensics-plots: Ohio 2004 President



(a) original data



(b) county-residualized data

Note: scatterplots, 2D empirical densities and marginal histograms for turnout and leader vote proportions. $n = 11364$ precincts.

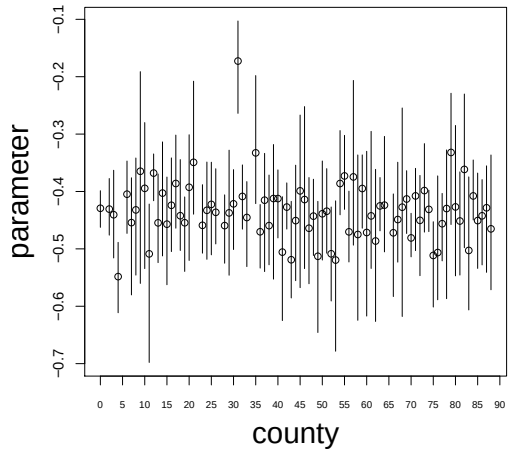
Table: Ohio 2004 President Election eforensics Estimates, County Fixed Effects

Type	Parameter	Covariate	Mean	lo[a]	up[b]
mixture probabilities	π_1	No Fraud	.774	.762	.788
	π_2	Incremental Fraud	.225	.210	.237
	π_3	Extreme Fraud	.000702	.000149	.00137
units eforensics-fraudulent: (1924 incremental, 5 extreme, 9435 not fraudulent)					
manufactured votes		$F_t = 79378.2\ [73570.1, 83534.4]^e$			
incremental manufactured		$F_t = 78920.2\ [73082.0, 83153.5]^e$			
extreme manufactured		$F_t = 458.0\ [281.5, 550.1]^e$			
total eforensics-fraudulent votes		$F_w = 177874.2\ [166201.6, 186814.6]^e$			
incremental total		$F_w = 176753.1\ [165035.9, 185860.4]^e$			
extreme total		$F_w = 1121.1\ [710.5, 1329.1]^e$			

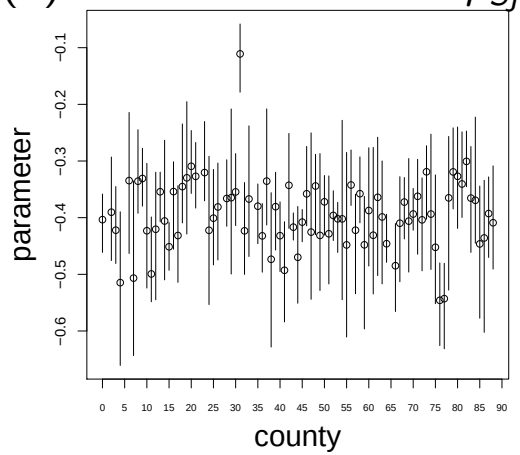
Note: selected eforensics model parameter estimates (posterior means and credible intervals). County fixed effects for turnout, vote choice and eforensics-frauds magnitudes are not shown. $n = 11364$ precinct units. Electors, valid votes and votes for the leader: $\sum_{i=1}^{n} N_i = 7972292$; $\sum_{i=1}^{n} V_i = 5411161$; $\sum_{i=1}^{n} W_i = 2766860$. [a] 95% HPD lower bound. [b] 95% HPD upper bound. [e] posterior mean [99.5% credible interval].

Figure: Ohio 2004 President: eforensics-frauds Magnitude Fixed Effect Parameters

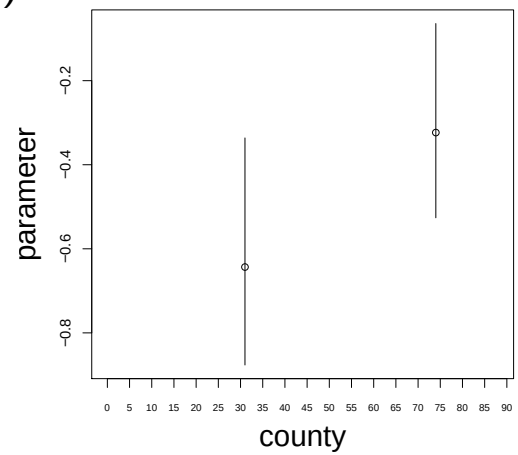
(a) incremental manufactured: ρ_{Mj}



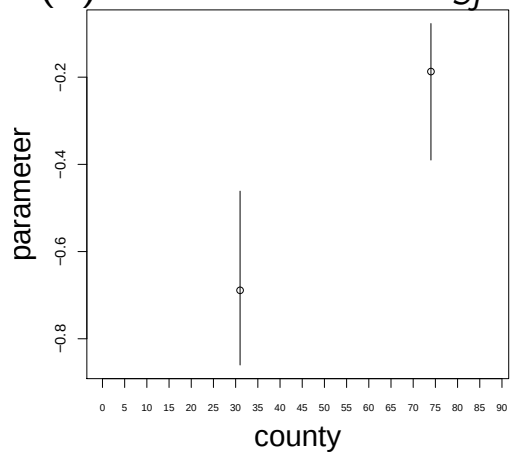
(b) incremental stolen: ρ_{Sj}



(c) extreme manufactured: δ_{Mj}

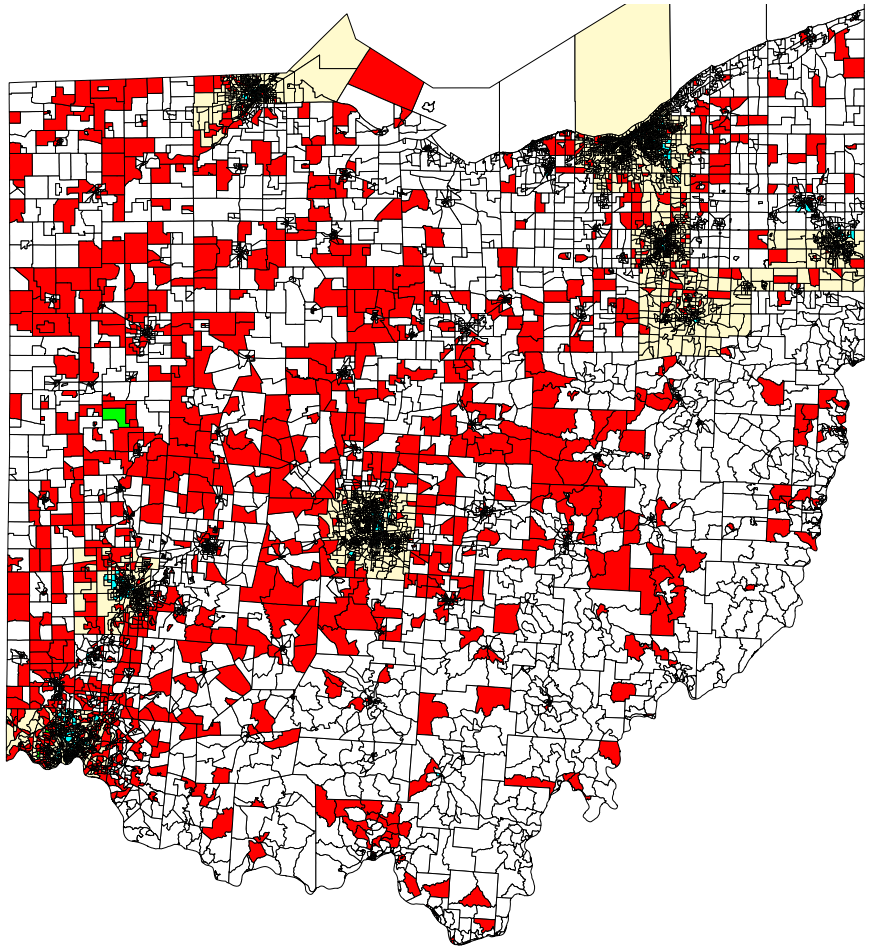


(d) extreme stolen: δ_{Sj}



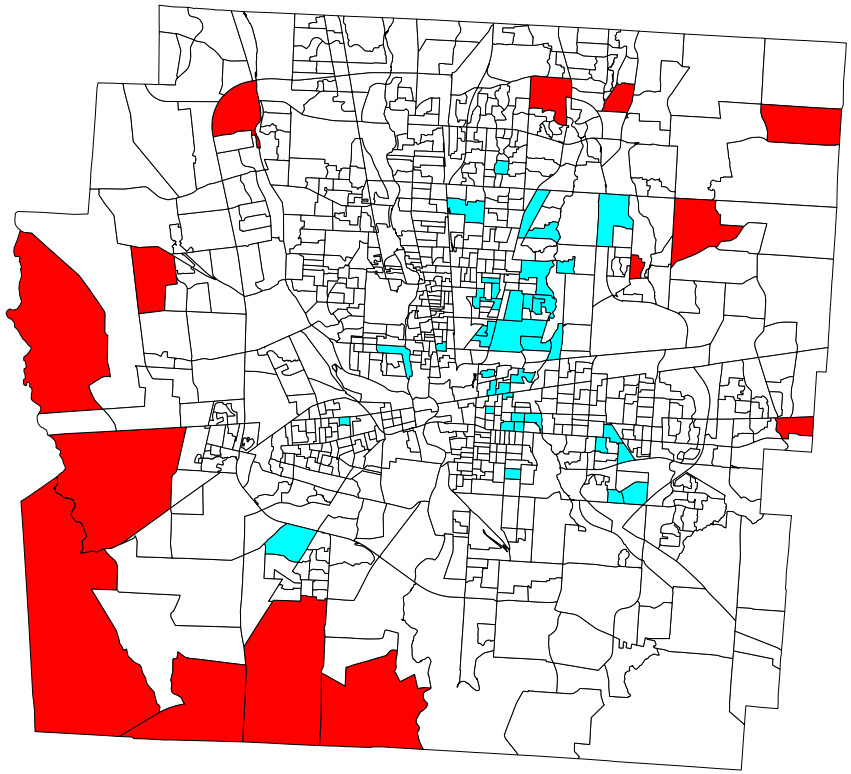
Note: active fixed effects parameters (posterior means and 95% HPD intervals) for frauds magnitude (ρ_{Mj}, ρ_{Sj}, δ_{Mj}, δ_{Sj}) parameters Incremental frauds are active in

Figure: Ohio 2004 President eforensics-frauds



Note: 2004 eforensics estimates mapped into 2007 precinct boundaries. Precincts in Cuyahoga, Franklin, Hamilton, Lucas, Mahoning, Montgomery, Stark or Summit counties are lemonchiffon. Precincts that have eforensics-frauds are red, except three precincts with extreme frauds are green. Precincts where the proportion of

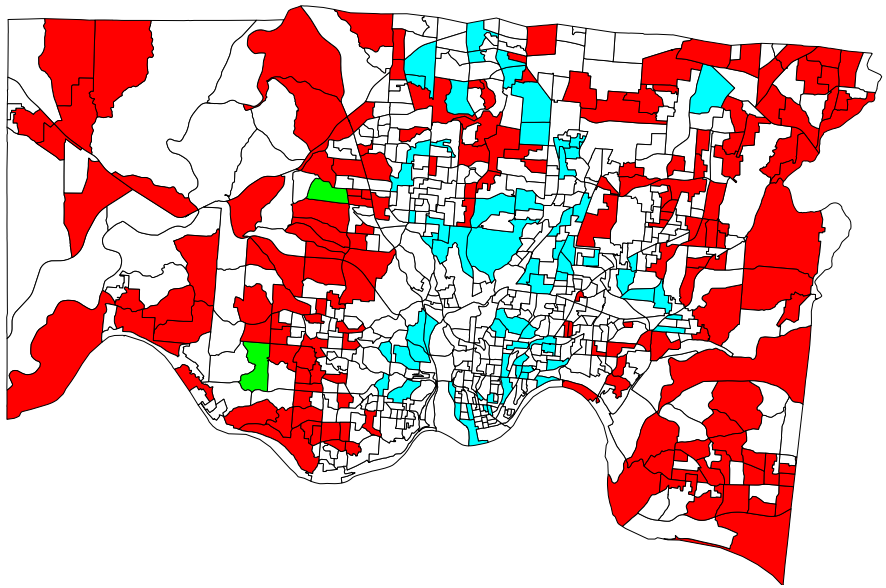
Figure: Ohio 2004 President eforensics-frauds, Franklin County



(a) Franklin County (main city Columbus)

Note: 2004 eforensics estimates mapped into 2007 precinct boundaries. Precincts where the proportion of electors who are African American is greater than .5 are cyan. Precincts that have eforensics-frauds are red.

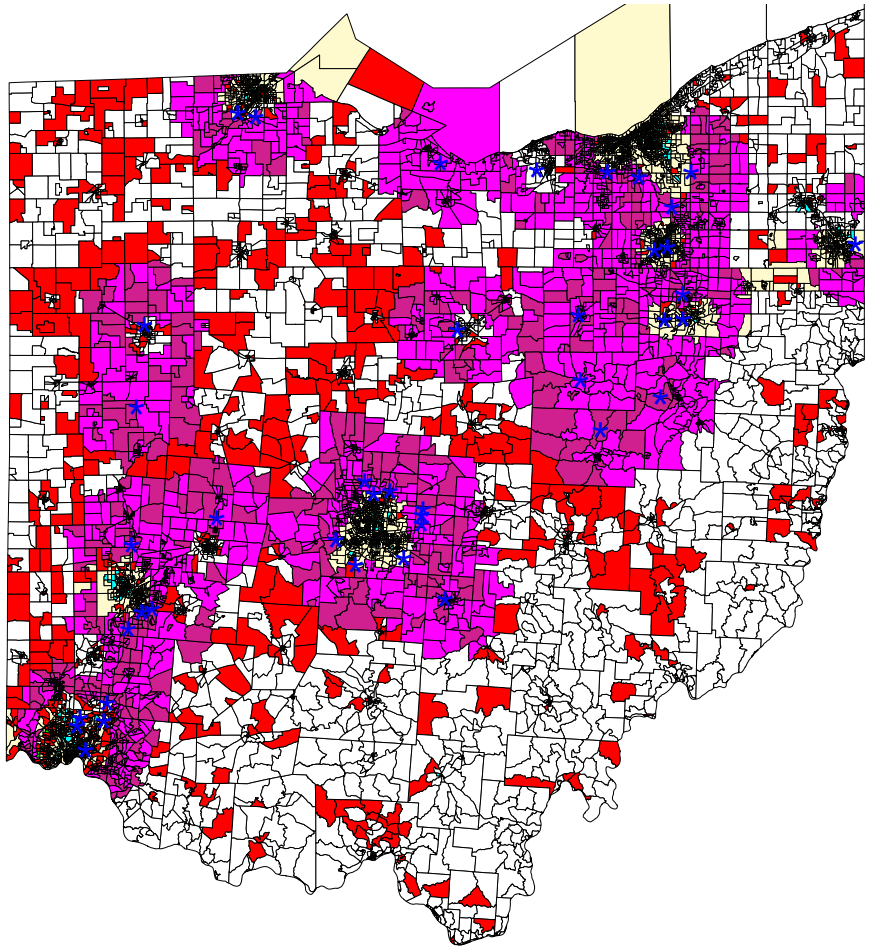
Figure: Ohio 2004 President eforensics-frauds, Hamilton County



(a) Hamilton Couny (main city Cincinnati)

Note: 2004 eforensics estimates mapped into 2007 precinct boundaries. Precincts where the proportion of electors who are African American is greater than .5 are cyan. Precincts that have eforensics-frauds are red, except two precincts with extreme frauds are green.

Figure: Ohio 2004 President eforensics-frauds with Megachurches



Note: 2004 eforensics estimates mapped into 2007 precinct boundaries. Megachurch locations ($n = 41$) are indicated by large blue asterisk characters. Precincts close to (within 16 miles of) a megachurch but not near (within 5 miles of) a precinct with a high proportion of African Americans are violetred if they have eforensics-frauds or

Table: Ohio 2004 President eforensics-frauds by Megachurch Close Status

	close to a megachurch?							
			if near high African American precinct					
eforensics-frauds?	(a) all MC		(b) skip		(c) omit		(d) only	
	no	yes	no	yes	no	yes	no	yes
no	.824	.880	.939	.776	.816	.776	.954	.911
yes	.176	.120	.0615	.224	.184	.224	.0471	.0891
n	2458	6896	6962	2392	2320	2392	3439	763
log-odds (SE)[a]	−.45 (.065)		1.48 (.070)		.24 (.073)		.68 (.150)	

Note: crosstable column percentages of eforensics-frauds presence in precincts by megachurch close status. (a) all precincts within 16 miles of a megachurch are "close." (b) "Close" is as in (a) except precincts within 16 miles of a megachurch but also within five miles of a precinct for which more than .5 of electors are African American are not counted as "close." (c) "Close" is as in (a) except precincts within five miles of a precinct for which more than .5 of electors are African American are omitted. (d) all precincts within 3 miles of a megachurch are "close," and only precincts within 4 miles of a precinct for which more than .5 of electors are African American are included.

[a] log-odds ratio for the relationship between a precinct's being close to a megachurch and its having eforensics-frauds.

Estimating Realized Election Frauds and eforensics

- eforensics-frauds and eforensics-fraudulent votes measure malevolent distortions of electors' intentions
- but eforensics can be ambiguous due to electors' strategic behaviors and lost votes (not addressed in this talk)
- the eforensics model has limitations beyond not directly dealing with the challenges of strategic behavior and lost votes (e.g., it can underestimate frauds, has frauds benefiting only one candidate, can be slow to compute estimates)
- nonetheless I am amazed that given only precinct (or other low aggregation unit) counts of electors, votes cast and leader votes, it is possible to produce valid estimates of the prevalence and magnitude of malevolent distortions of electors' intentions
- eforensics estimates are valid but imperfect

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